

GUJARAT TECHNOLOGICAL UNIVERSITY**B.E. Sem-IV Examination June- 2010****Subject code: 140001****Subject Name: Mathematics-4****Date: 15 / 06 / 2010****Time: 10.30 am – 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Do as directed.

(14)

- (a) Find the value of $\text{Re}(f(z))$ and $\text{Im}(f(z))$ at the indicated point where

$$f(z) = \frac{1}{1-z} \text{ at } 7+2i.$$

- (b) Find the value of the derivative of $\frac{z-i}{z+i}$ at i .

- (c) Find an upper bound for the absolute value of the integral $\int_C e^z dz$, where C is the line segment joining the points $(0,0)$ and $(1, 2\sqrt{2})$.

- (d) Evaluate $\oint_C \frac{dz}{z^2+1}$, where C is $|z+i|=1$, counterclockwise.

- (e) Develop $f(z) = \sin^2 z$ in a Maclaurin series and find the radius of convergence.

- (f) Define : (i) Singular point (ii) Essential singularity
(iii) Removable singularity (iv) Residue of a function

- (g) If $f(x) = \frac{1}{x}$, find the divided differences $[a,b]$ and $[a,b,c]$.

Q.2 (a) Evaluate $\int_0^1 e^{-x^2} dx$ by the Gauss integration formula with $n=3$. (03)

- (b) Compute $f(9.2)$ from the following values using Newton's divided difference formula. (04)

x	8	9	9.5	11.0
f(x)	2.079442	2.197225	2.251292	2.397895

- (c) (i) Find the positive root of $x = \cos x$ correct to three decimal places by bisection method. (03)

- (ii) Solve the following system of equations using partial pivoting by Gauss-elimination method. (04)

$$\begin{aligned} 8x_2 + 2x_3 &= -7 \\ 3x_1 + 5x_2 + 2x_3 &= 8 \\ 6x_1 + 2x_2 + 8x_3 &= 26 \end{aligned}$$

OR

- (c) (i) Find the dominant eigen value of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method and hence (03)

find the other eigen value also. Verify your results by any other matrix theory.

- (ii) Solve the following system of equations by Gauss- seidal method. (04)

$$10x_1 + x_2 + x_3 = 6$$

$$x_1 + 10x_2 + x_3 = 6$$

$$x_1 + x_2 + 10x_3 = 6$$

- Q.3 (a) Determine the interpolating polynomial of degree three using Lagrange's interpolation for the table below : (04)

x	-1	0	1	3
f(x)	2	1	0	-1

- (b) Evaluate $\int_0^3 \frac{dx}{1+x}$ with n=6 by using Simpson's $\frac{3}{8}$ rule and hence calculate (05)

log2. Estimate the bound of error involved in the process.

- (c) Using improved Euler's method, solve $\frac{dy}{dx} + 2xy^2 = 0$ with the initial condition y(0)=1 and compute y (1) taking h = 0.2. Compare the answer with exact solution. (05)

OR

- Q.3 (a) Find an iterative formula to find \sqrt{N} (where N is a positive number) and hence find $\sqrt{5}$. (04)

- (b) Compute cosh 0.56 from the following table and estimate the error. (05)

x	0.5	0.6	0.7	0.8
cosh x	1.127626	1.185465	1.255169	1.337435

- (c) Apply Runge-Kutta method of fourth order to calculate y (0.2) given $\frac{dy}{dx} = x+y$, y (0)=1 taking h=0.1 (05)

- Q.4 (a) Find and plot all roots of $\sqrt[3]{8i}$. (03)

- (b) Find out (and give reason) whether f (z) is continuous at z =0 if (03)

$$f(z) = \frac{\operatorname{Re}(z^2)}{|z|}, \quad z \neq 0$$

$$= 0, \quad z = 0$$

- (c) Using residue theorem, evaluate $\oint_C \frac{z^2 \sin z}{4z^2 - 1} dz$, $C : |z| = 2$ (04)

- (d) (i) Expand $f(z) = \frac{1-e^z}{z}$ in Laurent's series about z =0 and identify the singularity. (02)

- (ii) Find all solutions of $\sin z = 2$. (02)

OR

- Q.4 (a) Solve the equation $z^2 - (5+i)z + 8 + i = 0$. (03)

- (b) Show that if f(z) is analytic in a domain D and $|f(z)| = k = \text{const.}$ in D, then $f(z) = \text{const.}$ in D. (03)

- (c) Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center 0. (04)

- (d) (i) Find the center and the radius of convergence of the power series (02)

$$\sum_{n=0}^{\infty} (n+2i)^n z^n$$

(ii) State and prove Cauchy's residue theorem.

(02)

Q.5 (a) Find and sketch the image of region $x \geq 1$ under the transformation $w = \frac{1}{z}$ (03)

(b) Using the residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$ (03)

(c) Evaluate $\int_C \operatorname{Re}(z^2) dz$, where C is the boundary of the square with vertices (04)

0, i, $1 + i$, 1 in the clockwise direction.

(d) (i) State and prove Cauchy integral theorem. (02)

(ii) Determine a and b such that $u = ax^3 + bxy$ is harmonic and find a conjugate harmonic. (02)

OR

Q.5 (a) Define Möbius transformation. Determine the Möbius transformation that maps $z_1 = 0$, $z_2 = 1$, $z_3 = \infty$ onto $w_1 = -1$, $w_2 = -i$, $w_3 = 1$ respectively. (03)

(b) Using contour integration, show that $\int_0^\infty \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$ (03)

(c) Evaluate $\oint_C \frac{e^z}{z(1-z)^3} dz$, where C is (a) $|z| = \frac{1}{2}$ (b) $|z-1| = \frac{1}{2}$. (04)

(d) Check whether the following functions are analytic or not. (04)

(1) $f(z) = z^{\frac{5}{2}}$ (ii) $f(z) = \bar{z}$
